

NAG Toolbox for MATLAB

f07ca

1 Purpose

f07ca computes the solution to a real system of linear equations

$$AX = B,$$

where A is an n by n tridiagonal matrix and X and B are n by r matrices.

2 Syntax

```
[dl, d, du, b, info] = f07ca(dl, d, du, b, 'n', n, 'nrhs_p', nrhs_p)
```

3 Description

f07ca uses the Gaussian elimination with partial pivoting and row interchanges to solve the equations $AX = B$. The matrix A is factorized as $A = PLU$, where P is a permutation matrix, L is unit lower triangular with at most one nonzero subdiagonal element per column, and U is an upper triangular band matrix, with two superdiagonals.

Note that the equations $A^T X = B$ may be solved by interchanging the order of the arguments **du** and **dl**.

4 References

Anderson E, Bai Z, Bischof C, Blackford S, Demmel J, Dongarra J J, Du Croz J J, Greenbaum A, Hammarling S, McKenney A and Sorensen D 1999 *LAPACK Users' Guide* (3rd Edition) SIAM, Philadelphia URL: <http://www.netlib.org/lapack/lug>

5 Parameters

5.1 Compulsory Input Parameters

1: **dl(*)** – double array

Note: the dimension of the array **dl** must be at least $\max(1, n - 1)$.

Must contain the $(n - 1)$ subdiagonal elements of the matrix A .

2: **d(*)** – double array

Note: the dimension of the array **d** must be at least $\max(1, n)$.

Must contain the n diagonal elements of the matrix A .

3: **du(*)** – double array

Note: the dimension of the array **du** must be at least $\max(1, n - 1)$.

Must contain the $(n - 1)$ superdiagonal elements of the matrix A .

4: **b(ldb,*)** – double array

The first dimension of the array **b** must be at least $\max(1, n)$

The second dimension of the array must be at least $\max(1, \text{nrhs_p})$

Note: To solve the equations $Ax = b$, where b is a single right-hand side, **b** may be supplied as a one-dimensional array with length **ldb** = $\max(1, n)$.

The n by r right-hand side matrix B .

5.2 Optional Input Parameters

1: **n** – int32 scalar

Default: The dimension of the array **d**.

n , the number of linear equations, i.e., the order of the matrix A .

Constraint: $n \geq 0$.

2: **nrhs_p** – int32 scalar

Default: The second dimension of the array **b**.

r , the number of right-hand sides, i.e., the number of columns of the matrix B .

Constraint: **nrhs_p** ≥ 0 .

5.3 Input Parameters Omitted from the MATLAB Interface

ldb

5.4 Output Parameters

1: **dl**(*) – double array

Note: the dimension of the array **dl** must be at least $\max(1, n - 1)$.

If **info** ≥ 0 , **dl** contains the $(n - 2)$ elements of the second superdiagonal of the upper triangular matrix U from the LU factorization of A in **dl**(1), **dl**(2), ..., **dl**($n - 2$).

2: **d**(*) – double array

Note: the dimension of the array **d** must be at least $\max(1, n)$.

If **info** ≥ 0 , **d** contains the n diagonal elements of the upper triangular matrix U from the LU factorization of A .

3: **du**(*) – double array

Note: the dimension of the array **du** must be at least $\max(1, n - 1)$.

If **info** ≥ 0 , **du** contains the $(n - 1)$ elements of the first superdiagonal of U .

4: **b**(ldb,*) – double array

The first dimension of the array **b** must be at least $\max(1, n)$

The second dimension of the array must be at least $\max(1, \text{nrhs_p})$

Note: To solve the equations $Ax = b$, where b is a single right-hand side, **b** may be supplied as a one-dimensional array with length **ldb** = $\max(1, n)$.

If **info** = 0 or **info** $\geq N + 1$, the n by r solution matrix X .

5: **info** – int32 scalar

info = 0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

info = $-i$

If **info** = $-i$, parameter i had an illegal value on entry. The parameters are numbered as follows:

1: **n**, 2: **nrhs_p**, 3: **dl**, 4: **d**, 5: **du**, 6: **b**, 7: **ldb**, 8: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

info > 0

If **info** = i , u_{ii} is exactly zero, and the solution has not been computed. The factorization has not been completed unless $i = \mathbf{n}$.

7 Accuracy

The computed solution for a single right-hand side, $\hat{x}$, satisfies an equation of the form

$$(A + E)\hat{x} = b,$$

where

$$\|E\|_1 = O(\epsilon)\|A\|_1$$

and ϵ is the *machine precision*. An approximate error bound for the computed solution is given by

$$\frac{\|\hat{x} - x\|_1}{\|x\|_1} \leq \kappa(A) \frac{\|E\|_1}{\|A\|_1},$$

where $\kappa(A) = \|A^{-1}\|_1 \|A\|_1$, the condition number of A with respect to the solution of the linear equations. See Section 4.4 of Anderson *et al.* 1999 for further details.

Alternatives to f07ca, which return condition and error estimates are f04bc and f07cb.

8 Further Comments

The total number of floating-point operations required to solve the equations $AX = B$ is proportional to nr .

The complex analogue of this function is f07cn.

9 Example

```
d1 = [3.4;
      3.6;
      7;
      -6];
d = [3;
     2.3;
     -5;
     -0.9;
     7.1];
du = [2.1;
      -1;
      1.9;
      8];
b = [2.7;
     -0.5;
     2.6;
     0.6;
     2.7];
[d1Out, dOut, duOut, bOut, info] = f07ca(d1, d, du, b)

d1Out =
    -1.0000
     1.9000
     8.0000
    -6.0000
dOut =
     3.4000
```

```
      3.6000
      7.0000
     -6.0000
     -1.0154
duOut =
      2.3000
     -5.0000
     -0.9000
      7.1000
bOut =
     -4.0000
      7.0000
      3.0000
     -4.0000
     -3.0000
info =
      0
```
